ITERATIVE SOLUTION OF THE WAVEGUIDE-HORN JUNCTION

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Abstract

A solution for the fields at a waveguide-horn junction is derived using the iterative technique of overlapping regions. By dividing the junction into semi-infinite rectangular and sectoral regions and using the Green's functions for the sub-regions, the problem is reduced to the solution of a system of integral equations of the Fredholm type. The solution is relatively simple and the method is shown to be efficient and reliable in comparison with other techniques particularly for the reflection coefficient in the feed waveguide.

Introduction

It is generally known that the field expansions in the sectoral and rectangular sub-regions of a horn-waveguide junction cannot be directly matched because of the absence of a common boundary. Therefore, one has to either introduce a third field region in the segment, where neither expansion is valid[1] and which can be matched to both expansions in the two sub-regions, or to proceed to approximate solutions. An approximate solution, which is said to agree well with experiment for small sectoral flare angles, is obtained by matching the fields of the dominant mode in both sub-regions at one point $r = R$ and $\phi = 0$ in Fig. 1[2,3]. Generalized formulation, which treated such class of junction problems is given elsehwere: Rice[4] used a conformal mapping technique to obtain an approximate expression for the reflection coefficient for both TE and TM excitation, while Leonard and Yen[5] proposed an iterative procedure which becomes prohibitively complicated after more than two steps. Hamid[6], however, has obtained an approximate expression for the mode coefficients in the sectoral horn by ray theory.

In this paper, a solution of the uniform waveguide-horn junction is obtained using the iterative technique of overlapping regions.

Formulation

Schwarz's method of overlapping regions involves dividing the junction into semi-infinite wedge-shaped and rectangular overlapping sub-regions as shown in Fig. 1. Using the Green's functions for the two sub-regions, the problem is reduced to the solution of a system of integral equations of the Fredholm type which is solved by an alternating procedure as described below.

For the case of an H-plane junction with sectoral waveguide of flare angle α, we assume a TE_{10} mode excitation in the feed rectangular waveguide i.e. the non-zero electric field component is given by $E_y^i = A \sin(\frac{\pi x}{b})\, e^{ik_1 z}$ where $K_n = [K_o^2 - (\frac{n\pi}{b})^2]^{\frac{1}{2}}$, $K_o = 2\pi/\lambda$, λ is the wavelength, A is the amplitude of the incident field and the time dependence is taken as exp $(-i\omega t)$.

In order to formulate integral equations for the electric fields $E_{y_{I,II}}$ in sub-regions I and II, we make use of the Green's functions for Dirichlet boundary condition, namely $G^{(e)}_{I,II}$. Hence it is easy to show that

$$E_{y_I}(x,z) = A \sin(\frac{\pi x}{b})\, e^{ik_1 z} - \frac{1}{4\pi}\int_0^b \left[E_{y_I}(x_o,z_o)\, \frac{\partial G_I^{(e)}(x,z/x_o,z_o)}{\partial z_o}\right]\Bigg|_{z_o = 0} dx_o \qquad (1)$$

described with respect to origin 0, and

$$E_{y_{II}}(r,\phi) = -\frac{1}{4\pi}\int_0^R \left[E_{y_{II}} \cdot \frac{\partial G_{II}^{(e)}(\vec{r}/\vec{r}_o)}{r_o \partial\phi_o}\right]\Bigg|_{\phi_o = \frac{\alpha}{2}} dr_o + \frac{1}{4\pi}\int_0^R \left[E_{y_{II}} \cdot \frac{\partial G_{II}^{(e)}(\vec{r}/\vec{r}_o)}{r_o \partial\phi_o}\right]\Bigg|_{\phi_o = 2\pi - \frac{\alpha}{2}} dr_o \qquad (2)$$

described with respect to origin 0', where

$$G^{(e)}_{>I}(x,z/x_o,z_o) =$$

$$\frac{2}{b}\sum_{n=1}^{\infty} \frac{\sin(K_n z_o)\sin(n\pi x_o/b)\sin(n\pi x/b)\exp(-ik_n z)}{K_n}$$

$$G^{(e)}_{>II}(\vec{r}/\vec{r}_o) =$$

$$\frac{i\pi}{\alpha}\sum_{n=1}^{\infty} H_\nu^{(1)}(Kr)\, J_\nu(Kr_o)\cos\nu\phi\, \cos\nu\phi_o,\ \nu = (2n-1)\pi/\alpha$$

and where $G^{(e)}_{<I,II}$ are to be obtained from $G^{(e)}_{>I,II}$ by interchanging the source and observation point coordinates.

The second term in equation (1) requires integration over the boundary OSB since E_{y_I} vanishes over the metallic boundaries of region I and the first term is the contribution from the boundary at $z = -\infty$. It is also clear that in equation (2) the contribution from the boundary at $r = \infty$ is zero since the fields have to satisfy the radiation condition. For a symmetrical junction, the two terms in equation (2) add since the fields are equal at the boundaries $\phi_o = \frac{\alpha}{2}$ and $\phi_o = 2\pi - \frac{\alpha}{2}$.

We start the alternating procedure by first considering the semi-infinite rectangular region. Assuming values for $E_{y_I}\big|_{z_o = 0}$ (with amplitude related to A as will be seen later) on the boundary OSB and using equation (1), it is then possible to determine E_{y_I} at any point interior to region I, namely on the planes 00' and 0'B which are the boundaries of the wedge-shaped region and on which the field is considered in the integration of equation (2). Substituting these values in equation (2), it will be possible to determine the field at points interior to the sectoral sub-region, namely along the boundary OSB, which when substituted again in equation (1) leads to a second order approximation and so on.

Since the procedure requires that the assumed values for $E_{y_I}\big|_{z_o=0}$ must be normalized to A, the normalization equation is obtained by equating the fields in both sub-regions at a common boundary in the over-lapping region, such as OSB. In fact, it is easier to use this equation alternatively by calculating the amplitude of the incident field for an assumed $E_{y_I}\big|_{z_o=0}$ with unity amplitude and then to use the obtained A as normalizing constant. A final comment on the solution is that while $G_>(x,z/x_o,z_o)$ will be used in equation (1) (since the singular source distribution, which is equivalent to the non-zero fields on the boundaries[7] is on the boundary $z_o=0$), both $G^{(e)}_{\lessgtr II}$ are to be used in equation (2) to calculate the field on the boundary OSB. Since for every point $p(r,\phi)$ along OSB we calculate r_p and ϕ_p, therefore for $r<r_p$ we use $G^{(e)}_{<II}(\vec{r}/\vec{r}_o)$ and for $r>r_p$ we use $G^{(e)}_{>II}(\vec{r}/\vec{r}_o)$ and consequently each integral in equation (2) is divided into two parts.

If the incident field is the dominant TM mode with the only non-zero magnetic field component given by $H^i_y = e^{ikz}$, the formulation procedure is essentially the same as in the TE case except for the use of Green's functions $G^{(m)}_{I,II}$ corresponding to the Neumann boundary condition. In this case there will appear under the integral sign of the equations corresponding to (1) and (2) the function $G^{(m)}_{I\ or\ II}$ multiplied by the derivative of the magnetic fields H_y with respect to a unit normal pointing outward from the surface. To overcome the infinitely large values of this derivative at the junction edges,we exclude the singularity using the least square curve fitting technique[8].

Although the initially assumed fields on the boundaries OSB may be completely arbitrary, a reasonably good guess from physical considerations will be necessary to reduce the number of iterations. The Green's function of the semi-infinite rectangular sub-region suggests that a Fourier sine series would be quite appropriate even though a constant field distribution i.e. $E_{y_I}(x_o,z_o)\big|_{z_o=0}$ = constant is allowed by the method as a last resort.

While the convergence of the successive approximation to the correct solution is discussed elsewhere[9], the method has also proved to be efficient since the computation time is quite adequate compared with other numerical techniques[10-12]. In particular, the method is found to be more efficient for similar geometries than the finite difference algorithm[13] and fairly comparable with the straight-forward and extended point matching techniques[14]. However, a major limitation of the method is the required computation time demanded by many numerical integrations[12] particularly to match both field expansions on the common boundary OSB. In order to reduce the computation time, and hence improve the overall efficiency of the method, we consider matching the two field expansions on a common boundary of a sub-region of the overlapping region. This is achieved by considering sub-triangles of the overall triangle O'OSB, by shifting the base from $z = 0$ to $z = -d$ where d varies from zero to $R\cos\frac{\alpha}{2}$. Thus in the limit when d equals zero the previous results are obtained, while in the other limit when $d = R\cos\frac{\alpha}{2}$ the matching boundary shrinks to a single point coinciding with the apex of the sectoral waveguide, hence allowing comparison with published results for the magnitude of the reflection coefficient obtained by matching the dominant mode in both regions at the mid-point of the junction[1,2]. Between these two limits the results for junction reflection coefficient are presented together with the required computation time and compared with published data[15-17].

Conclusions

The iterative procedure involving overlapping regions has been applied to the junction of rectangular and sectoral waveguides to obtain efficient solution for the field distribution and junction reflection coefficient which compare favourably with published results based on other techniques. These results can be used to select a matching element at the junction and to obtain the overall reflection coefficient at the junction of a sectoral horn and a rectangular feed waveguide[18]. Scattering matrix techniques can also be used to obtain the solution for higher order modes or multiple junctions which can be similarly formulated by the present method.

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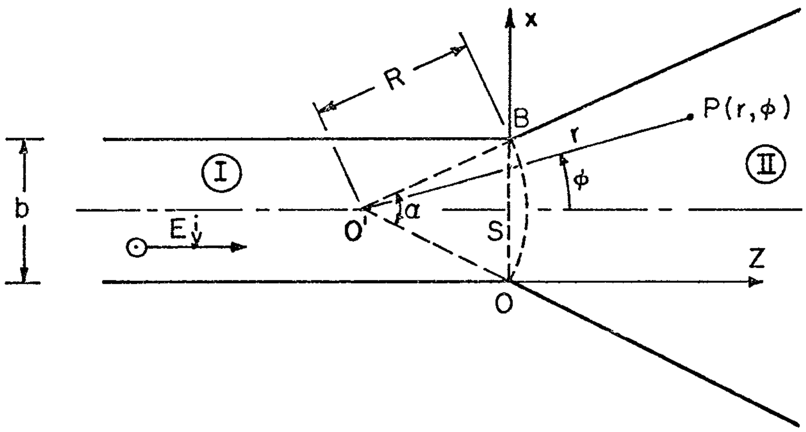

Fig. 1